Violation of the cosmic no hair conjecture in Einstein-Maxwell-dilaton system

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Abstract

The cosmic no hair conjecture is tested in the spherically symmetric Einstein-Maxwell-dilaton (EMD) system with a positive cosmological constant Λ. Firstly, we analytically show that once gravitational collapse occurs in the massless dilaton case, the system of field equations breaks down inevitably in outer communicating regions or at the boundary provided that a future null infinity $\mathcal{I}^+$ exists. Next we find numerically the static black hole solutions in the massive dilaton case and investigate their properties for comparison with the massless case. It is shown that their Abbott-Deser (AD) mass are infinite, which implies that a spacetime with finite AD mass does not approach a black hole solution after the gravitational collapse. These results suggest that $\mathcal{I}^+$ cannot appear in the EMD system once gravitational collapse occurs and hence the cosmic no hair conjecture is violated in both the massless and the massive cases, in contrast to general relativity.

I. INTRODUCTION

Recent cosmological observations [1] suggest that there must be a positive cosmological constant Λ in our universe. Furthermore, it is widely believed that the inflation took place in the early stage of our universe and the vacuum energy of a scalar field plays a role of Λ efficiently. In such a spacetime, most regions expand exponentially as in de Sitter spacetime. However, when the inhomogeneity of the initial matter distribution is very high, some regions would collapse into a black hole unless the inhomogeneous region is too large [2], or the Abbott-Deser (AD) mass is negative [3]. Such a spacetime is classified as an *asymptotically de Sitter (ASD) spacetime* [4], where there exists a de Sitter-like spacelike future null infinity $\mathcal{I}^+$.

Gibbons and Hawking [5] proposed the cosmic no hair conjecture, which states that every spacetime with nonzero Λ approaches Kerr-Newman-de Sitter spacetime in the stationary limit. From a different point of view, the conjecture at least seems to state that most regions expand exponentially and the future null infinity $\mathcal{I}^+$ should appear even though gravitational collapse occurs somewhere. Although the proof of the conjecture is only shown in a restricted class of spacetimes [6], it is widely believed that any spacetime with Λ results in ASD spacetime after gravitational collapse in general relativity.

What seems to be lacking, however, is the picture of the gravitational collapse in a spacetime with Λ in the framework of generalized theories of gravity. It is important to take some corrections into account when we consider the dynamics in spacetime regions with high curvature. The purpose of this paper is to investigate the gravitational collapse and to test the cosmic no hair conjecture in an effective string theory, which is one of promising generalized theories of gravity. Especially, we consider the Einstein-Maxwell-dilaton (EMD) system, which naturally arises from a low energy limit of string theory [7].

Poletti et al. [8] found that the EMD system with massless dilaton has no static spherically symmetric black hole solution in ASD spacetime. This is essentially due to the fact that there is no regular configuration of the dilaton field between a black hole event horizon (BEH) and a cosmological event horizon (CEH) satisfying the boundary conditions. Thus, one may naively expect that gravitational collapse cannot occur in the EMD system and hence no black holes appear even though highly inhomogeneous region exists. However, this is not likely because the EMD system satisfies the dominant energy condition, which implies that a highly inhomogeneous region would continue to collapse. So, what is the final state of the collapse in such a system? Motivated by this, we investigate the dynamics of the spherically symmetric EMD system when the gravitational collapse occurs.

The rest of this paper is organized as follows. In Sec. II we write down the basic equations and set the initial conditions. In Secs. III and IV we analytically show that the field equations of the EMD system with massless dilaton inevitably break down in the domain of outer communicating regions or at the boundary provided that there exists a null infinity $\mathcal{I}^+$. Here, we present the detailed proof of our previous result [9]. In Sec. V, we find the static black hole solutions in the EMD system with massive dilaton. It is shown that they are stable for linear perturbations but their AD mass are infinite. In Sec. VI we discuss the cosmic no hair conjecture in the EMD system on the basis of our results.

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II. BASIC EQUATIONS AND INITIAL VALUE CONDITIONS

The action in the EMD system with a positive cosmological constant Λ (> 0) is

$$S = \int d^4x\sqrt{-g}\,[-R + 2(\nabla\phi)^2 + 2V_\phi + e^{-2a\phi}F^2 + 2\Lambda], \quad (2.1)$$

where V_ϕ and a represent the potential and the coupling constant of the dilaton field ϕ, respectively. String theory requires $a = 1$. Varying the action (2.1), we obtain the field equations

$$\nabla_\mu(e^{-2a\phi}F^{\mu\nu}) = 0, \quad (2.2)$$

$$\nabla^2\phi + \frac{a}{2}e^{-2a\phi}F^2 - \frac{1}{2}\frac{\partial V_\phi}{\partial\phi} = 0, \quad (2.3)$$

$$\begin{aligned} R_{\mu\nu} &= 2\nabla_\mu\phi\nabla_\nu\phi + (V_\phi + \Lambda)g_{\mu\nu} \\ &\quad + 2e^{-2a\phi}F_{\mu\rho}F_\nu{}^\rho - \frac{1}{2}g_{\mu\nu}e^{-2a\phi}F^2. \end{aligned} \quad (2.4)$$

In the spherically symmetric system, the Maxwell equation (2.2) is automatically satisfied for a purely magnetic Maxwell field $F = Q\sin\theta d\theta\wedge d\phi\,(F^2 = 2Q^2/R^4)$, where Q is a magnetic charge and R is a circumference radius. Because an electrically charged solution is obtained by a duality rotation from the magnetically charged one [10,11], we have only to consider the purely magnetic case.

It is worth noting that we cannot take any regular initial data (S, h_{ab}, K_{ab}) in the EMD system because the field strength of the Maxwell field F^2 diverges at $R = 0$, where h_{ab} is the metric on a 3-dimensional spacelike hypersurface S embedded in (M, g) and K_{ab} is the extrinsic curvature. Physically, it seems reasonable to suppose that the existence of this central charge results from the gravitational collapse of an appropriate charged matter field such as a charged perfect fluid, as depicted in Fig. 1. In that case, we can take regular initial data (S, h_{ab}, K_{ab}).

Once gravitational collapse occurs, a closed trapped surface $\mathcal{T}$ will appear because the matter field falls into a small region. As shown in the proof of Ref. [4], $\mathcal{T}$ cannot be seen from $\mathcal{I}^+$ and hence a BEH inevitably appears in ASD spacetime [1]. It follows that the ASD spacetime has a null characteristic hypersurface N whose boundary is a closed future-trapped surface in the late stage of the gravitational collapse (see Fig. 1). For simplicity, we make the physical assumption that all the charged matter fields fall into the BEH in the past of N. In the next section, we will investigate the evolution of the field equations (2.2)-(2.4) from N analytically under the assumption that spacetime becomes ASD spacetime.

[1]We should note that ASD spacetime excludes the possibility that a naked singularity appears. See Ref. [4] in detail.

III. ANALYSIS OF THE DYNAMICS OF MASSLESS DILATON FIELD

In this section, we analytically investigate the evolution of the system from N in the case of massless dilaton field, i.e. $V_\phi = 0$ and present a theorem which states that the evolution from any initial data on N results in the breakdown of the field equations in outer communicating regions or at the boundary in ASD spacetime. Since we are interested in effective superstring theory, hereafter we put $a = 1$.

This section is divided into two parts. Firstly, we shall state some reasonable assumptions and present the theorem. As a first step to the proof, we next consider the asymptotic behavior of the dilaton field on both event horizons.

A. Assumptions and Theorem

Firstly, we assume that surface gravity of both horizons are almost constant asymptotically. By using Gaussian null coordinates covering the BEH (CEH),

$$\begin{aligned} ds^2 &= -2dr\,d\eta + F\,(r,\eta)d\eta^2 \\ &\quad + R^2(r,\eta)(d\theta^2 + \sin^2\theta d\phi^2), \end{aligned} \quad (3.1)$$

let us define the surface gravity of the BEH (CEH) κ_B (κ_C) by $F_{,r}/2|_{r=0}$, where ∂_r is a future-directed ingoing (outgoing) null geodesic intersecting the BEH (CEH) and $F = r = 0$, $F_{,r} > 0$ on the BEH (CEH). If $(\partial_\eta)^\mu$ is a timelike killing vector field in $r < 0$, each surface gravity reduces to the usual surface gravity κ defined as

$$\nabla^\mu(\eta^\nu\eta_\nu) = -2\kappa\,\eta^\mu. \quad (3.2)$$

As shown in [5], each surface gravity and each event horizon area corresponds to the temperature and the entropy of the corresponding event horizon if the space-time is almost static. In addition, it has been shown that the areas of both BEH and CEH cannot decrease and have an upper bound, $12\pi/\Lambda$ [4,12]. Therefore, it is clear that both areas approach positive constants asymptotically. This physically implies that in a neighborhood of each horizon, the spacetime would approach locally a static one. On the analogy of the almost static spacetime, we shall assume

$$\lim_{\eta\to+\infty}\kappa_B \sim \text{const.} > 0, \quad (3.3)$$

and

$$\lim_{\eta\to+\infty}\kappa_C \sim \text{const.} > 0, \quad (3.4)$$

in ASD space-time. The above assumptions indicate that if the entropy of the BEH (CEH) is asymptotically constant, its temperature neither goes to 0 (this is the third law of event horizon [13]) nor diverges.

Next we shall assume that the outgoing null expansion of the CEH does not approach 0 asymptotically. Let us consider Gaussian null coordinates covering the CEH and an outgoing null geodesic, ∂_r intersecting the CEH. Defining the outgoing null expansion $\theta_+ \equiv 2R_{,r}/R$, the Raychaudhuri equation (see Ref. [14]) is

$$\frac{d\theta_+}{dr} = -\frac{1}{2}{\theta_+}^2 - \phi_{,r}^2. \tag{3.5}$$

This means that θ_+ cannot become negative or 0, otherwise, the outgoing null geodesic would not reach the null infinity $\mathcal{I}^+$ ($R = \infty$). Therefore, we also assume

$$\inf_\eta \theta_C(\eta) > 0, \qquad \eta \in [0, +\infty), \tag{3.6}$$

or equivalently

$$\inf_\eta R_{,r}(\eta)|_C > 0, \qquad \eta \in [0, +\infty), \tag{3.7}$$

because the area of the CEH, R_C has an upper bound. Here, θ_C is the expansion at the CEH. This assumption means that θ_+ at the CEH cannot approach 0 in the future.

Under the above assumptions we will consider the evolution of the field equations (2.3), (2.4) in the massless dilaton case. In terms of double null coordinates,

$$\begin{aligned} ds^2 = &-2\, e^{-\lambda}\,(U,V)\, dU dV \\ &+ R(U,V)^2(d\theta^2 + \sin^2\theta d\phi^2). \end{aligned} \tag{3.8}$$

The field equations (2.3), (2.4) can be reduced to the dynamical field equations

$$2R^3\,(R_{,U}\phi_{,V} + R\phi_{,UV} + R_{,V}\phi_{,U}) = Q^2 e^{-2\phi-\lambda}, \tag{3.9}$$

$$\lambda_{,UV} - \frac{2R_{,UV}}{R} = 2\phi_{,U}\phi_{,V} + e^{-\lambda}\left(\frac{Q^2 e^{-2\phi}}{R^4} - \Lambda\right), \tag{3.10}$$

$$R_{,UV} + \frac{R_{,U}R_{,V}}{R} = -\frac{e^{-\lambda}}{2R}\left(1 - \frac{Q^2 e^{-2\phi}}{R^2} - \Lambda R^2\right), \tag{3.11}$$

and the constraint equations

$$R_{,UU} + \lambda_{,U}R_{,U} = -(\phi_{,U})^2 R, \tag{3.12}$$

$$R_{,VV} + \lambda_{,V}R_{,V} = -(\phi_{,V})^2 R, \tag{3.13}$$

where $A_{,X}$ is a partial derivative of A with respect to X.

We present the following theorem.

Theorem 1 *Let us consider the dynamical evolution of the equations (3.9)-(3.11) with initial data on the characteristic null hypersurface N in ASD spacetime. Then, there is a $U_1\,(\leq U_B)$ such that the system of equation breaks down at $U = U_1$ in the sense that the equations cannot be evolved from the hypersurface N to the surface U_B, where U_B is the coordinate at the BEH.*

B. Asymptotic behavior of the dilaton field

We investigate the asymptotic behavior of the dilaton field and give the following lemma.

Lemma 1 *The asymptotic values of the dilaton field ϕ cannot diverge at any of the two event horizons. To be precise,*

$$\sup_\eta |\phi(0,\eta)| < +\infty, \qquad \eta \in [0, +\infty) \tag{3.14}$$

must be satisfied in ASD spacetime.

To prove this, let us consider Gaussian null coordinates (3.1) covering the CEH and estimate a quantity $I(\eta)$ defined on each outgoing null hypersurface ($\eta =$ const. null hypersurface),

$$I(\eta) = \int_{-r_N(\eta)}^{0} \phi_{,r}^2(r,\eta)\, dr, \tag{3.15}$$

where $-r_N(\eta)$ is the value of r at the intersection between N and $\eta =$ const. null hypersurface. Because the area of the CEH has an upper bound, $12\pi/\Lambda$, $r_N(\eta)$ also has an upper bound as follows,

$$\begin{aligned} 0 < r_N(\eta) = \int_{R_N(\eta)}^{R_C(\eta)} \frac{2dR}{\theta_+ R}\bigg|_\eta &< \int_{R_N(\eta)}^{R_C(\eta)} \frac{2dR}{\theta_C R_N}\bigg|_\eta \\ &< C(R_C(\eta) - R_N(\eta)) < C\sqrt{\frac{3}{\Lambda}}, \quad C(>0), \end{aligned} \tag{3.16}$$

where θ_C, R_C, and R_N are the values of θ_+, R at the CEH and the value of R at N, respectively [2]. Therefore, we can obtain

$$\lim_{\eta\to\infty} I(\eta) = +\infty \quad \text{when} \quad \lim_{\eta\to\infty}\phi(0,\eta) = \pm\infty \tag{3.17}$$

because the following inequality

$$\begin{aligned} 2|\phi(0,\eta) - \phi(-r_N,\eta)| - r_N(\eta) &\leq \int_{-r_N(\eta)}^{0} (2|\phi_{,r}| - 1)|_\eta\, dr \\ &\leq \int_{-r_N(\eta)}^{0} {\phi_{,r}}^2|_\eta dr = I \end{aligned} \tag{3.18}$$

is satisfied and $\phi(-r_N,\eta)$ has a limit at the intersection of N and the BEH,

$$\lim_{\eta\to\infty}\phi(-r_N,\eta) = \phi|_{\rm BEH}. \tag{3.19}$$

[2] We derived the first inequality by using the fact that θ_+ is a positive and strictly decreasing function in the outer regions of the BEH by Eq. (3.5). To derive the second inequality, we used the assumption (3.6).

Then, because of the Raychaudhuri equation (3.5), θ_C behaves as

$$\lim_{\eta\to\infty} \theta_C < -\lim_{\eta\to\infty} I = -\infty. \tag{3.20}$$

This is a contradiction because θ_+ must be positive in the outer regions of the BEH. Similarly, we can also see that ϕ on the BEH should not diverge asymptotically. Thus, Eq. (3.14) must be satisfied at both event horizons. □
Under the above observation, we will prove theorem 1.

IV. PROOF OF THE THEOREM

We shall prove theorem 1 by contradiction below. In the first step, we consider the asymptotic behavior of field functions near the CEH. It is convenient to rescale the coordinate U in the double null coordinates (3.8) such that U is an affine parameter of a null geodesic of the CEH, i.e, λ is constant along the CEH. Hereafter we use the character u $(= F(U))$ instead of U in this parameterization to avoid confusion. Under such coordinates, Eq. (3.12) on the CEH is

$$R_{,uu} = -(\phi_{,u})^2 R \leq 0. \tag{4.1}$$

By the assumption (3.4), u is asymptotically related to the coordinate η of Eq. (3.1) by

$$u \sim e^{\kappa_C \eta} \quad \text{on the CEH.} \tag{4.2}$$

Therefore, the null geodesic generators of the CEH are future complete in asymptotically de Sitter space-time. Because the area of the CEH is non-decreasing and has an upper bound,

$$\lim_{u\to+\infty} R_{,u} = 0 \quad \text{and} \quad \lim_{u\to+\infty} R = C_1 \tag{4.3}$$

along the CEH, where C_i $(i = 1, 2, ..)$ is a positive constant. This implies that $R_{,uu}(\leq 0)$ on the CEH must converge to 0 faster than u^{-2}. Hence the future asymptotic behavior of $R_{,uu}$ on the CEH is represented as follows,

$$\lim_{u\to+\infty} u^2 R_{,uu} = -\lim_{u\to+\infty} f(u) = 0, \tag{4.4}$$

where $f(u)$ is some non-negative function. Then, by Eq. (4.1) and lemma 1

$$\phi_{,u} \sim u^{-1}\sqrt{f}, \qquad \text{and} \qquad \lim_{u\to+\infty} \phi = \text{const.} \tag{4.5}$$

We shall obtain the asymptotic value of $R_{,V}$ on the CEH by solving Eq. (3.11) as

$$R_{,V} = \left(\int_{u_i}^{u} \frac{KR}{R_i} du + R_{,V}|_i\right) \frac{R_i}{R}, \tag{4.6}$$

$$K = -\frac{e^{-\lambda}}{2R}\left(1 - \frac{Q^2 e^{-2\phi}}{R^2} - \Lambda R^2\right),$$

where $R_{,V}|_i$ and R_i are initial values of $R_{,V}$ and R on $u = u_i$, respectively. K must approach a positive constant K_∞ as $u \to +\infty$ because R, $\phi \to$ const. by lemma 1 and the expansion at the CEH behaves as $\lim_{\eta\to+\infty} \theta_+ \sim \lim_{\eta\to+\infty} u^{-1} R_{,V} > 0$ by Eq. (3.6). Then, the asymptotic value of $R_{,V}$ is

$$R_{,V} \sim K_\infty u > 0. \tag{4.7}$$

We shall also obtain the asymptotic value of $\phi_{,V}$ on the CEH by solving Eq. (3.9) and the solution is the following,

$$\phi_{,V} = \left(\int_{u_i}^{u} \frac{HR}{R_i} du + \phi_{,V}|_i\right) \frac{R_i}{R}, \tag{4.8}$$

$$H = \frac{Q^2 e^{-2\phi-\lambda}}{2R^4} - \frac{R_{,V}\,\phi_{,u}}{R},$$

where $\phi_{,V}|_i$ is an initial value of $\phi_{,V}$ on $u = u_i$. H approaches a positive constant H_∞ as $u \to \infty$ because $R_{,V}\phi_{,u} \sim \sqrt{f} \to 0$ by Eqs. (4.5) and (4.7). The asymptotic value of $\phi_{,V}$ is

$$\phi_{,V} \sim H_\infty u > 0. \tag{4.9}$$

Let us denote a timelike hypersurface $r = -\epsilon$ by T_C, where r is the affine parameter of an outgoing null geodesic ∂_r in Gaussian null coordinates and ϵ is a small positive constant. If we take ϵ small enough, an infinitesimally small neighborhood $\mathcal{U}_C$ of the CEH contains T_C. Let us denote each point of the intersection of T_C and $u =$ const. hypersurface by $p(u)$. By using Eq. (3.9), the solution of $h \equiv \phi_{,u}$ along each $u =$ const. is

$$h = \left(-\int_{V}^{V_C} \frac{LR}{R_C} dV + h_C\right) \frac{R_C}{R}, \tag{4.10}$$

$$L = \frac{Q^2 e^{-2\phi-\lambda}}{2R^4} - \frac{R_{,u}\phi_{,V}}{R}, \tag{4.11}$$

where R_C and h_C are values of R and h on the CEH, $V = V_C$, respectively. Let us consider the asymptotic value $(R_{,u}\phi_{,V})|_{V_C}$. By Eqs. (4.1), (4.5), and (4.9)

$$\lim_{u\to+\infty} R_{,u}\phi_{,V} \sim \lim_{u\to+\infty} u \int_u^\infty u'^{-2} f\, du'$$

$$= \lim_{u\to+\infty} \frac{u^{-2} f}{u^{-2}} = \lim_{u\to+\infty} f = 0, \tag{4.12}$$

where we used l'Hospital's rule in the first equality. Therefore, $L \sim L_\infty > 0$ asymptotically. Differentiating h by V in Eq. (4.10),

$$h_{,V} = L + \frac{R_C R_{,V}}{R^2}\left(\int_V^{V_C} \frac{LR}{R_C} dV - h_C\right). \tag{4.13}$$

By using Eq. (4.2) and the relation between Gaussian null coordinates (3.1) and double null coordinates (3.8), we obtain

$$\epsilon \sim u\,\delta V \quad \text{on the CEH} \tag{4.14}$$

for large u. By the relation (4.14) for large values of u, $h_{,V} \sim L_\infty + O(\epsilon) + O(\sqrt{f})$ on T_C. $h_{,V} \sim L_\infty > 0$ on the $u =$ const. null segment $[V|_{p(u)}, V_C] \subset \mathcal{U}_C$ asymptotically, since ϵ is an arbitrary small value. Here we obtain the result in the first step as $h_{,V} \sim L_\infty > 0$.

In the next step we investigate the behavior of $\phi, \phi_{,V}$, and R on the BEH, just like in the CEH case. We rescale V into v such that v is an affine parameter and $\lambda =$ const. on the BEH, while we leave U unchanged. The relation between v and η of Eq. (3.1) is given by

$$v \sim e^{\kappa_B \eta} \quad \text{on the BEH,} \tag{4.15}$$

as given in the CEH case (4.2). Because the area of the BEH is also non-decreasing and it has an upper bound, as in the case of the CEH,

$$\lim_{v\to+\infty} R_{,v} = 0 \quad \text{and} \quad \lim_{v\to+\infty} R = C_2. \tag{4.16}$$

Therefore we have

$$\lim_{v\to+\infty} v^2 R_{,vv} = -\lim_{v\to+\infty} g(v) = 0, \tag{4.17}$$

along the BEH, where $g(v)$ is some non-negative function. This means that by Eq. (3.13)

$$\phi_{,v} \sim v^{-1}\sqrt{g}, \qquad \text{and} \qquad \lim_{v\to\infty} \phi = \text{const.} \tag{4.18}$$

at the BEH. By replacing u by v in the argument of the CEH case and solving Eqs. (3.9) and (3.11), the asymptotic behaviors of $R_{,U}$ and $\phi_{,U}$ become

$$R_{,U} \sim v, \qquad \phi_{,U} \sim C_3\, v. \tag{4.19}$$

Hence the first and third terms in the l.h.s. of the dilaton field Eq. (3.9) are negligible asymptotically and then

$$\lim_{v\to+\infty} k_{,U} = \lim_{v\to+\infty} \phi_{,vU} = C_4 > 0. \tag{4.20}$$

Here we define $k \equiv \phi_{,v}$. Let us consider an infinitesimally small neighborhood $\mathcal{U}_B$ of the BEH. There is a small positive ϵ such that a timelike hypersurface T_B with $r = -\epsilon$ is contained in $\mathcal{U}_B$, where r is the affine parameter of an ingoing null geodesic ∂_r in Gaussian null coordinates (3.1). By using the relation (4.15), we obtain

$$\epsilon \sim v\,\delta U. \tag{4.21}$$

k on T_B is asymptotically

$$\begin{aligned} k|_{T_B} &\sim k|_{BEH} + k_{,U}|_{BEH}\ (-\delta U) \\ &\sim k|_{BEH} - C_4\,\epsilon\, v^{-1} \\ &\sim -(\epsilon\, C_4 + \sqrt{g})\ v^{-1}. \end{aligned} \tag{4.22}$$

Now, we reached the result in the second step, i.e.,$\phi_{,V}$ is negative on T_B for large values v $(> v_1)$.

Let us consider each $u =$ const. null segment $N_u(u \geq u_I) : [V_C - \epsilon/u_I, V_C]$, where $h_{,V} \sim L_\infty > 0$ on N_{u_I} (see Fig. 2). If one takes u_I large enough, N_u intersects $T_B(v > v_1)$ at $u = u_F$. Let us take a sequence of $N_{u_J}(J = 1, 2, ..., L+1)$ (L is a natural number large enough), where $\delta u = (u_F - u_I)/L$ and $u_J = u_I + (J-1)\,\delta u$. Hereafter we denote N_{u_J} as N_J. Before proving theorem 1, we shall establish the following two lemmas.

Lemma 2 *$R_{,u} < 0$ on each $V =$ const. null segment ($V_C - \epsilon/u_I \leq V < V_C$) between the BEH and T_C.*

Proof. Let us consider Eq. (3.11). The solution is given by

$$R_{,u} = \left(-\int_V^{V_C} \frac{KR}{R_C} dV + R_{,u}|_C \right) \frac{R_C}{R}. \tag{4.23}$$

Substituting $V = V_C - \epsilon/u$, $R_{,u}$ on T_C is asymptotically

$$\begin{aligned} R_{,u}(V = V_C - \epsilon/u) &\sim -\epsilon K_\infty u^{-1} + R_{,u}|_C \\ &\sim -\epsilon K_\infty u^{-1} + O(f)\, u^{-1}. \end{aligned} \tag{4.24}$$

This means that if we take u_I large enough, $R_{,u} < 0$ on T_C for any $u \geq u_I$ and hence $R_{,u} < 0$ on each $V =$ const. ($V_C - \epsilon/u_I \leq V < V_C$) null segment between the BEH and T_C by Eq. (3.12). □

Let us consider a function $Q^2 e^{-2\phi-\lambda}$ on a compact region, $V_C - \epsilon/u_I \leq V \leq V_C$, $u_I \leq u \leq u_F$ and take the minimum $Q^2 e^{-2\phi-\lambda}|_{min} > 0$. Defining h_{min} by $h_{min} \equiv \min\{L_\infty, Q^2 e^{-2\phi-\lambda}|_{min}\}$, we give a lemma below.

Lemma 3 *$h_{,V} \geq h_{min} > 0$ on N_J if $\phi_{,V} > 0$ on N_J.*

Proof. Let us consider each $u = u_J$ null segment: $[V_C - \epsilon/u_J, V_C] \in \mathcal{U}_\mathcal{C}$, $[V_C - \epsilon/u_I, V_C - \epsilon/u_J] \notin \mathcal{U}_\mathcal{C}$. In the former null segment, $h_{,V} \geq h_{min} > 0$, as already discussed before. In the latter null segment, $\int_V^{V_C} (LR/R_C)\, dV \geq \int_{V_C-\epsilon/u_J}^{V_C} (LR/R_C)\, dV \sim \epsilon/u_J \gg h_C$ because $L > 0$ by Eq. (4.10) and the previous lemma. This indicates that $h_{,V} \geq h_{min} > 0$ on N_J by Eq. (4.13). □

Now, we can prove Theorem 1. By the above lemma we can expand $\phi_{,V}|_{N_{J+1}}$ by δu such that $\phi_{,V}|_{N_{J+1}} \cong (\phi_{,V}|_{N_J} + h_{,V}|_{N_J}\delta u) \geq (\phi_{,V}|_{N_J} + h_{min}\,\delta u)$ because δu is arbitrary small and $h_{,V}$ has a positive lower bound, h_{min}. This means that $\phi_{,V}|_{N_{J+1}} > \phi_{,V}|_{N_J} > 0$. On the other hand, $\phi_{,V} > 0$ on $u = u_I$ by Eq. (4.9), hence $\phi_{,V} > 0$ for each u_J by induction. This is a contradiction because $\phi_{,V} < 0$ on $T_B(u = u_F)$ as shown before. Therefore, the assumption that field equations (3.10)-(3.13) could be evolved until $U = U_B$ is false. □

V. MASSIVE DILATON CASE

In this section, we examine the system with the massive dilaton field. The unexpected properties in the massless dilaton case mainly comes from the fact that there is no static black hole solution which the system would approach asymptotically after the gravitational collapse [8]. Hence we look for the static solutions in the massive dilaton case for the first step.

Here, we employ the potential $V(\phi) = 2m_\phi^2 e^{2\phi}\phi^2$, where m_ϕ is the mass of the dilaton field. We are not sure whether this form of the potential is an exact one or not. However for small perturbation of the dilaton away from its vacuum value, we might expect a quadratic form to be a good approximation. Moreover, the existence of the static solution does not depend on the detail of the potential form if it is locally convex. Actually the results we will show are not changed qualitatively for $V(\phi) = 2m_\phi^2\phi^2$.

A. static solutions

We assume the following static chart metric

$$ds^2 = -f(t,r)e^{-\delta(t,r)}dt^2 + f(t,r)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \qquad (5.1)$$

$$f(t,r) = 1 - \frac{2m(t,r)}{r} + \frac{Q^2}{r^2} - \frac{\Lambda}{3}r^2. \qquad (5.2)$$

Then the field equations (2.3) and (2.4) are expressed as

$$-\left[e^{\delta}f^{-1}\dot{\phi}\right]^{\cdot} + \frac{1}{r^2}\left[r^2e^{-\delta}f\phi'\right]' = e^{-\delta}\left[m_\phi^2e^{2\phi}\phi(1+\phi) - \frac{e^{-2\phi}Q^2}{r^4}\right], \qquad (5.3)$$

$$m' = \frac{r^2}{2}\left[e^{2\delta}f^{-1}\dot{\phi}^2 + f\phi'^2 + m_\phi^2e^{2\phi}\phi^2 + \frac{(e^{-2\phi}-1)Q^2}{r^4}\right], \qquad (5.4)$$

$$\delta' = -r\left[e^{2\delta}f^{-2}\dot{\phi}^2 + \phi'^2\right], \qquad (5.5)$$

$$\dot{m} = r^2f\dot{\phi}\phi', \qquad (5.6)$$

where a prime and a dot denote derivatives with respect to the radial and time coordinates, respectively. Here, we have normalized variables and parameters by Λ as $\sqrt{\Lambda}t \to t$, $\sqrt{\Lambda}r \to r$, $\sqrt{\Lambda}m \to m$, $\sqrt{\Lambda}Q \to Q$ and $\sqrt{\Lambda}m_\phi \to m_\phi$. First we neglect the terms including time derivative for a while and look for the nontrivial static solutions.

For the boundary conditions, each functions should be finite in the $r_B \le r \le r_C$ for regularity. We can take $\delta(r_C) = 0$ without loss of generality. If we are interested in some different boundary condition, we can always have such a boundary condition by only rescaling the time coordinate.

The equation of the dilaton field (5.3) is rewritten as

$$e^{-\delta}f\phi'' + \frac{1}{r^2}\left[r^2e^{-\delta}f\right]'\phi' = e^{-\delta}\left[m_\phi^2e^{2\phi}\phi(1+\phi) - \frac{e^{-2\phi}Q^2}{r^4}\right]. \qquad (5.7)$$

Since $f = 0$ on the horizons, ϕ' is expressed by ϕ as

$$\phi' = \frac{1}{f'}\left[m_\phi^2e^{2\phi}\phi(1+\phi) - \frac{e^{-2\phi}Q^2}{r^4}\right]\Bigg|_{\text{horizon}}. \qquad (5.8)$$

We choose $\phi(r_B)$ and integrate from the BEH to the CEH. For some value $\phi(r_B) = \phi_1$, the dilaton field diverges to plus infinity. If we choose larger value $\phi(r_B) = \phi_2 > \phi_1$, it diverges to minus infinity. Then there is a value $\phi_* \in (\phi_1, \phi_2)$ with which the dilaton field becomes finite everywhere between the BEH and the CEH. In this sense $\phi(r_B)$ is a shooting parameter of this system, which must be determined by the iterative method.

In the massless case, ϕ decreases around the BEH because f' is positive, i.e., the right hand side (r.h.s.) in Eq. (5.7) is negative. Similarly it is locally increasing function around the CEH because $f' < 0$. On the other hand, at the extremum of the dilaton field ($\phi' = 0$), $\phi'' < 0$ since $f > 0$ for $r_B < r < r_C$. Hence the dilaton field does not have the minimum in this region. This contradict the behavior around the both horizons. As a result, there is no spherically symmetric static black hole solution in the massless dilaton case. In the massive dilaton case, however, the situation is different. As we can see, the mass term in Eq. (5.7) appears in the opposite sign to the charge term. Hence the sign of the r.h.s is determined by the value of these two terms.

Integrating the equations numerically, we found nontrivial solutions for some parameters m_ϕ, Q and r_B. We show the configurations of the dilaton field with $m_\phi = 0.1$, $Q = 0.4$ and several value of r_B in Fig. 3. The left (right) end point of each line corresponds to the BEH (CEH). The dilaton field decreases monotonically between the BEH and the CEH. This implies that the term including the magnetic charge in Eq. (5.3) is dominant near the BEH while the dilaton mass term becomes dominant around the CEH. The change of dominant contribution terms is essential for the existence of the static solutions in the massive dilaton case.

We find solutions with different values of m_ϕ and Q. There are three horizons for the Reissner-Nordström-de Sitter (RNdS) solution in some parameters, M and Q. Note that since all three horizons degenerate when $Q = 0.5$, there is no regular RNdS solution for $Q > 0.5$. From our analysis, new solutions seem not to have inner Cauchy horizon and we also find the solutions with

$Q > 0.5$ unlike the RNdS case. This is due to the suppression effect of the magnetic charge by the dilaton field.

B. stability analysis

Even if the static solutions exist, such objects do not exist in the physical situation if they are unstable. Hence we will investigate the stability of these new solutions. In this paper, we consider only the radial perturbations around the static solutions.

We expand the field functions around the static solution ϕ_0, m_0 and δ_0 as follows:

$$\phi(t,r) = \phi_0(r) + \frac{\phi_1(t,r)}{r}\epsilon, \tag{5.9}$$

$$m(t,r) = m_0(r) + m_1(t,r)\epsilon, \tag{5.10}$$

$$\delta(t,r) = \delta_0(r) + \delta_1(t,r)\epsilon. \tag{5.11}$$

Here ϵ is an infinitesimal parameter. Substituting them into the field functions (5.3)-(5.6) and dropping the second and higher order terms of ϵ, we find

$$\begin{aligned} & -e^{\delta_0} f_0^{-1}\ddot{\phi}_1 + \left[e^{-\delta_0} f_0 \phi_1'\right]' - \left\{\frac{1}{r}\left(e^{-\delta_0} f_0\right)' \right. \\ & +2re^{-\delta_0}\left[m_\phi^2 e^{2\phi_0}\phi_0(1+\phi_0) - \frac{e^{-2\phi_0}Q^2}{r^4}\right]\phi_0' \\ & \left. +e^{-\delta_0}\left[m_\phi^2 e^{2\phi_0}(1+2\phi_0) + \frac{2e^{-2\phi_0}Q^2}{r^4}\right]\right\}\phi_1 \\ & -\left[\frac{2}{r}\left(re^{-\delta_0}\phi_0'\right)' - 2re^{-\delta_0}\phi_0'^3\right] m_1 = 0, \end{aligned} \tag{5.12}$$

$$\dot{m}_1 = r^2 f_0 \phi_0' \dot{\phi}_1, \tag{5.13}$$

where $f_0 = 1 - 2m_0/r + Q^2/r^2 - r^2/3$. Next we set

$$\phi_1 = \xi(r)e^{i\sigma t}, \qquad m_1 = \eta(r)e^{i\sigma t}. \tag{5.14}$$

If σ is real, ϕ oscillates around the static solution and the solution is stable. On the other hand, if the imaginary part of σ is negative, the perturbation ϕ_1 and m_1 diverges exponentially with time and then the solution is unstable. By Eq. (5.13), the relation between ξ and η is

$$\eta = r f_0 \phi_0' \xi. \tag{5.15}$$

The perturbation equation of the scalar field becomes

$$-\frac{d^2\xi}{dr_*^2} + U(r)\xi = \sigma^2\xi, \tag{5.16}$$

where we employ the tortoise coordinate r_* defined by

$$\frac{dr_*}{dr} = e^{\delta_0} f_0^{-1}, \tag{5.17}$$

and the potential function is

$$\begin{aligned} U(r) = {} & e^{-\delta_0} f_0 \left\{\frac{1}{r}\left(e^{-\delta_0} f_0\right)' \right. \\ & +2re^{-\delta_0}\left[m_\phi^2 e^{2\phi_0}\phi_0(1+\phi_0) - \frac{e^{-2\phi_0}Q^2}{r^4}\right]\phi_0' \\ & +e^{-\delta_0}\left[m_\phi^2 e^{2\phi_0}(1+2\phi_0) + \frac{2e^{-2\phi_0}Q^2}{r^4}\right] \\ & \left. +2f_0\phi_0'\left[\left(re^{-\delta_0}\phi_0'\right)' - r^2 e^{-\delta_0}\phi_0'^3\right]\right\}. \end{aligned} \tag{5.18}$$

Being similar to the other variables, the eigenvalue σ^2 and the potential function U are normalized as $\sigma^2/\Lambda \to \sigma^2$ and $U/\Lambda \to U$.

Fig. 4 shows the potential functions $U(r)$ of the solution with $m_\phi = 0.1$ and $r_B = 0.5$. Since $d^2\xi/dr^{*2} = U(r) = 0$ on both horizons for the negative mode, ξ must approach zero as $r^* \to \pm\infty$ by the regularity of Eq. (5.16). Under this boundary condition we have searched for the negative eigenmodes. For the existence of the negative eigenmode, the depth of the potential is important. However, the potentials in Fig. 4 have rather shallow well and we can find no negative mode for any Q. Hence we conclude that the static solutions presented before are stable at least the radial perturbation level.

C. asymptotic structure

Finally, we will investigate the asymptotic structure of the static black hole solutions. From the analysis of static solutions, we obtain the boundary value of the field functions at the CEH. By use of these values we can now investigate the asymptotic behavior of the field functions for $r \to \infty$, which is expected to approach de Sitter spacetime. Fig. 5 shows the field configurations beyond the CEH. We can find that the dilaton field decreases to its potential minimum $\phi = 0$. Its decay rate is, however, extremely small. In the asymptotic region, the field equation of the dilaton field (5.3) behaves as

$$r^2\phi'' + 4r\phi' = \frac{m_\phi^2}{\lambda}\phi, \tag{5.19}$$

where we assume $f \sim -\lambda r^2$. If the mass function behaves as $m(r)/r^3 \to 0$ as $r \to \infty$, $\lambda = 1/3$. Putting $\phi \sim r^{-\alpha}$, Eq. (5.19) becomes

$$\alpha(\alpha+1) - 4\alpha = \frac{m_\phi^2}{\lambda}. \tag{5.20}$$

Hence we find

$$\alpha = \frac{3 - \sqrt{9 - 4m_\phi^2/\lambda}}{2}. \tag{5.21}$$

By the fact that the decay rate of the dilaton field is very small, the minus sign was taken. Moreover, in the $m_\phi = 0.1$ case, we can approximate as

$$\alpha = \frac{m_\phi^2}{3\lambda} \sim 10^{-2}. \tag{5.22}$$

This coincides with the behavior in Fig. 5. The equation of the mass function (5.4) behaves as

$$\begin{aligned} m' &= \frac{r^2}{2}\left(-\lambda r^2 \phi'^2 + m_\phi^2 \phi^2\right) \\ &= \frac{1}{2}\left(-\lambda\alpha^2 + m_\phi^2\right) r^{2-\alpha} \approx \frac{m_\phi^2}{2} r^2. \end{aligned} \tag{5.23}$$

Hence

$$m \sim \frac{m_\phi^2}{6} r^3. \tag{5.24}$$

This implies that the contribution of the dilaton field to the mass function is similar to the cosmological constant and it diverges as $r \to \infty$. Hence the AD mass diverges and this behavior is out of line with the ASD spacetime [4]. As a result, the massive dilaton field should also break the asymptotic structure similarly to the massless case.

VI. CONCLUDING REMARKS

We first tested the cosmic no hair conjecture in an effective string theory by investigating the dynamics of the EMD system with massless dilaton. We have shown that once gravitational collapse occurs, the system of the field equations inevitably breaks down in the domain of outer communicating regions or at the boundary under the existence of a de Sitter-like future null infinity $\mathcal{I}^+$.

In general, the breakdown of the field equations in the EMD system can be interpreted as follows: (a) a naked singularity appears in outer communicating regions or at the boundary [15] or (b) no initial null hypersurface N evolves into $\mathcal{I}^+$. For the first case, however, it is hard to understand that a naked singularity inevitably appears in any case because the dilaton and the electromagnetic fields are essential ingredient for the string theory and they are physically reasonable matter fields for testing the cosmic censorship conjecture [16]. Thus, we strongly expect that only case (b) is possible and hence the EMD system with massless dilaton violates the cosmic no hair conjecture [17]. Here, we should not overlook that our result is independent of the quantity of the collapsing mass.

The above result seems to arise from the fact that the system has no static spherically symmetric black hole solution [8] and also the dilaton field is non-minimally coupled to the electromagnetic field. Let us imagine the gravitational collapse in the Einstein-Maxwell (EM) system for comparison. Because all matter fields are minimally coupled to the electromagnetic field in the EM system, they fall into a black hole or escape from the CEH without difficulty and the resultant spacetime would asymptotically approach RNdS one. This implies that ASD spacetime appears in the EM system and the cosmic no hair conjecture holds, in contrast to the EMD system.

We next investigated the EMD system with massive dilaton and found that the system has a static black hole solution which is stable for radial linear perturbations. At first glance, this seems to imply that spacetime settles down to the solution after the gravitational collapse and the cosmic no hair conjecture holds in the massive dilaton case. However, as shown in Sec. V, the quasi-local mass diverges as $r \to \infty$. This means that AD mass also diverges because the quasi-local mass corresponds to the AD mass in static spherically symmetric spacetime. In this sense, this solution is unphysical. In addition, if we consider the dynamical evolution from regular initial data with finite AD mass, the spacetime seems not to approach our new solutions because AD mass is conserved during evolution. This would cause the problem again that the cosmic no hair conjecture is violated in the EMD system, which suggests that the massless dilaton case is *not* special.

As a result, we conclude that once gravitational collapse occurs in the spherically symmetric EMD system, the spacetime cannot approach ASD spacetime, in contrast to the EM system. To the best of our knowledge, this is the first counterexample for the cosmic no hair conjecture if the cosmic censorship holds. It is an open question whether by considering axially symmetric spacetime or more general spacetimes one could avoid this problem.

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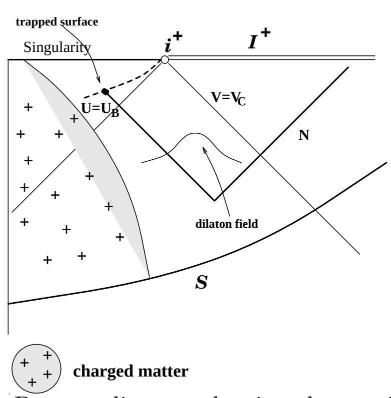

FIG. 1. A Penrose diagram showing the gravitational collapse of a charged matter fluid accompanied by a dilaton field supposing that ASD spacetime appears. A closed trapped surface is formed as the gravitational collapse proceeds. N is a characteristic null hypersurface.

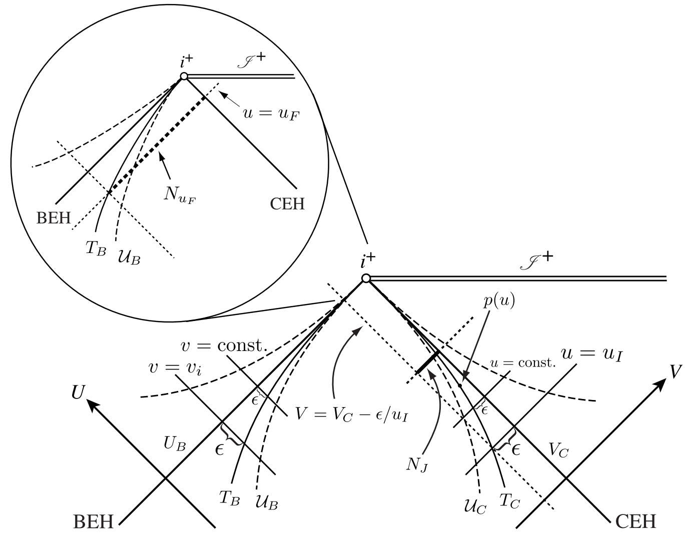

FIG. 2. Timelike hypersurfaces T_B, T_C are displayed in the neighborhoods $\mathcal{U}_B$, $\mathcal{U}_C$ of the BEH and the CEH, respectively. A null segment N_J is displayed by a thick line. ϵ is a fixed affine parameter distance along outgoing and ingoing null geodesics.

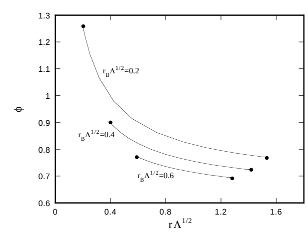

FIG. 3. The configurations of the dilaton field of the static black hole solutions with $m_\phi\sqrt{\Lambda} = 0.1$, $Q\sqrt{\Lambda} = 0.4$ and $r_B\sqrt{\Lambda} = 0.2$, 0.4, 0.6. The left (right) dots correspond to the BEH (CEH).

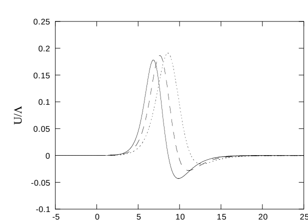

FIG. 4. The potential functions of the perturbative equation. We set $m_\phi\sqrt{\Lambda} = 0.1$, $r_B\sqrt{\Lambda} = 0.5$ and $Q\sqrt{\Lambda} = 0.2$ (solid line), 0.4 (dashed line), 0.6 (dotted line).

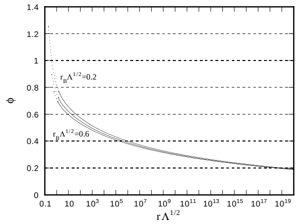

FIG. 5. The configurations of the dilaton field of the static solutions beyond the CEH. The parameters are same as in Fig. 3. The dotted part shows inside of the CEH. We find that the dilaton fields decays extremely slowly.